

MATHEMATICS

Chapter 11: Mensuration



Mensuration

Area of polygons

- Area of a rectangle = Length \times Breadth
 - Area of a square = (Side)²
 - Area of a triangle = $\frac{1}{2} \times$ Base \times Height
 - Area of a parallelogram = Base \times Height
 - Area of a trapezium = $\frac{1}{2}$ sum of parallel sides \times distance between them
 - Area of a rhombus = $\frac{1}{2} \times$ product of its diagonals
 - Area of a general quadrilateral can be found by dividing it into two triangles, by drawing on of its diagonals, and then applying the formula of area of a triangle.
1. Area of a polygon (or field) can be calculated by suitably dividing it into triangle, rectangle, trapezium etc.
 2. Surface area of a solid is the sum of the areas of all its faces.
 3. Amount of region occupied by a solid is called its volume.
 4. For a cuboid of length l , breadth b and height h , we have:
 - Volume of cuboid = $(l \times b \times h)$ cubic units
 - Total surface area of cuboid = $(lb + bh + lh)$ sq units
 - Lateral surface area of cuboid = $\{2(l + b) \times h\}$ sq units
 - Diagonal of cuboid $\sqrt{l^2 + b^2 + h^2}$ units
 5. For a cube of side a , we have:
 - Volume of cube = (a^3) cubic units
 - Total surface area of cube = $(6a^2)$ sq units
 - Lateral surface area of cube = $(4a^2)$ sq units
 - Diagonal of cube = $\sqrt{3a}$ units
 6. For a cylinder of height h and base radius r , we have:
 - Volume of cylinder = $(\pi r^2 h)$ cubic units
 - Curved surface area of cylinder = $(2\pi r h)$ sq units
 - Total surface area of cylinder = $2\pi r(h + r)$ sq units

- Unit conversion:
- $1 \text{ cm}^3 = 1 \text{ mL}$
- $1 \text{ L} = 1000 \text{ cm}^3$
- $1 \text{ m}^3 = 10^6 \text{ cm}^3 = 1000 \text{ L}$

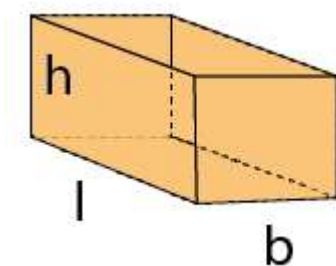
Mensuration

Mensuration is a branch of mathematics which mainly deals with the study of different kinds of Geometrical shapes along with its area, length, volume and perimeters. It is completely based on the application of both algebraic equations and geometric calculations. The results obtained by the Mensuration are considered very accurate. There are two types of geometric shapes:

Volume of a 3D Object

Volume is the space occupied by the three-dimensional object. It is a three-dimensional quantity.

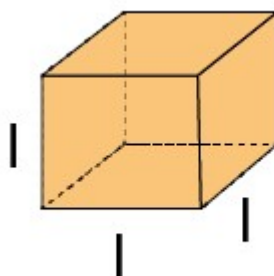
Volume of a Cuboid



$$\text{Volume of a cuboid} = l \times b \times h$$

where, l is the length, b is the breadth and h is the height of the cuboid.

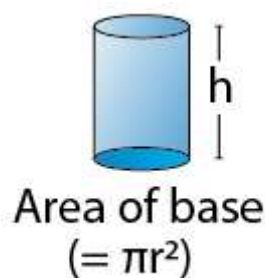
Volume of a Cube



$$\text{Volume of a cube} = l^3$$

Where, l is the length of each side of the cube.

Volume of a Cylinder



$$\text{Volume of the cylinder} = \pi r^2 h$$

Where r is the radius of the base and h is the height of the cylinder.

Basics Revisited

Mensuration is the study of geometry that deals with the measurement of length, areas and volumes.

Perimeter is the total length or path of a given shape.

Area is the total region covered by the given shape.

Volume is the total space occupied by the given shape.

Identifying Shapes and Areas of Different Regular Figures

Area of a Rectangle: length \times breadth, perimeter: 2 (length + breadth)

Area of Square: side \times side, perimeter: 4 \times side

Area of Triangle: $\frac{1}{2}$ (base \times height), perimeter: $a + b + c$ (sum of 3 sides)

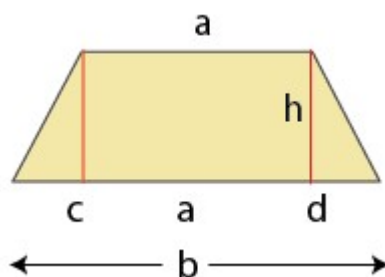
Area of Parallelogram: base \times height, perimeter: 2 (length + breadth)

Area of Circle: $\pi \times (\text{radius})^2$, perimeter: $2 \times \pi \times \text{radius}$.

Trapezium

Area of Trapezium by Division into Shapes of Known Area

Consider the trapezium where a and b are parallel sides, h is the height. Trapezium is divided into 3 parts: two triangles, one rectangle.



Here h is the height, a and b are 2 parallel sides.

Area of trapezium = Area of 2 triangles + Area of rectangle

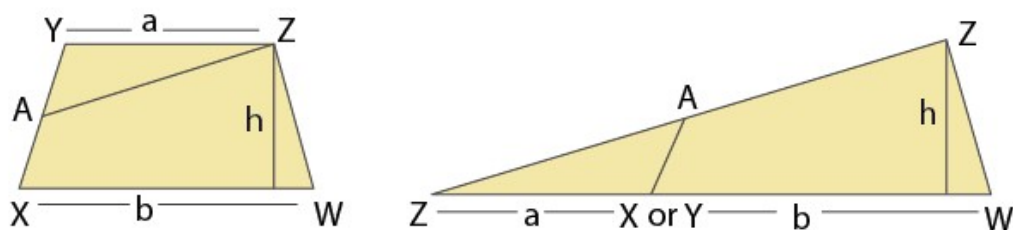
$$= \left[\frac{1}{2} \times c \times h + \frac{1}{2} \times d \times h \right] + [a \times h]$$

Area of Trapezium by Finding the Area of a Triangle of Same Area

The area of the trapezium can be found out by dividing it into a triangle and a polygon.

Consider a trapezium WXYZ. Mark a midpoint A for side XY and join AZ. Cut the trapezium along AZ and obtain a ΔAZY

Flip the ΔAZY and place it as shown below. Now the new polygon is a triangle.



We know that,

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

Substituting the values we get,

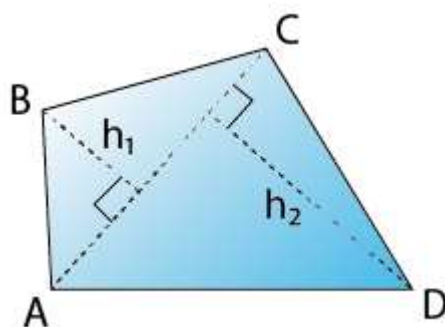
$$\text{Area of a triangle} = \frac{1}{2} \times (a + b) \times h$$

But the original polygon is a trapezium. So,

$$\text{Area of a trapezium} = \frac{1}{2} \times (a + b) \times h$$

Area of a General Quadrilateral

Consider a quadrilateral ABCD. Draw diagonal AC. From B and D draw perpendiculars h_1 , h_2 to AC



Area of quadrilateral = Area of triangle ABC + Area of triangle ADC

$$= \frac{1}{2} \times \text{base} \times \text{height} + \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \left(\frac{1}{2} \times AC \times h_1 \right) + \left(\frac{1}{2} \times AC \times h_2 \right) \text{ [Where, } h_1, h_2 \text{ are the heights, AC is the base]}$$

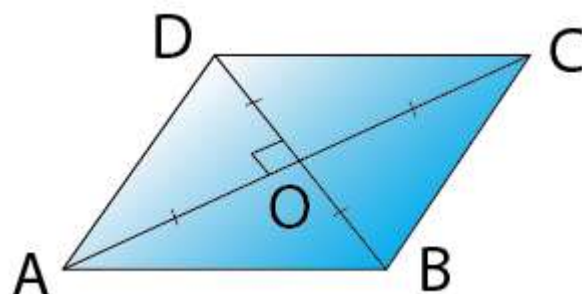
$$= \frac{1}{2} \times AC \times (h_1 + h_2) = \frac{1}{2} \times d \times (h_1 + h_2) \text{ [}\because \text{ AC is a diagonal]}$$

$$\therefore \text{Area of a Quadrilateral} = \frac{1}{2} \times d \times (h_1 + h_2)$$

where d is diagonal and h_1, h_2 are perpendicular drawn to a diagonal.

Area of Rhombus

$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2,$$

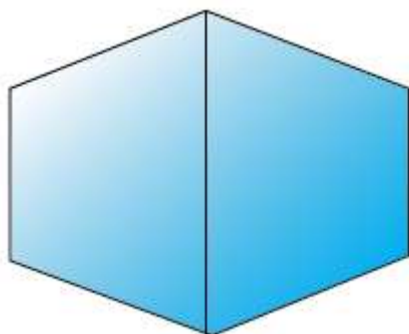


where d_1 and d_2 are the diagonals.

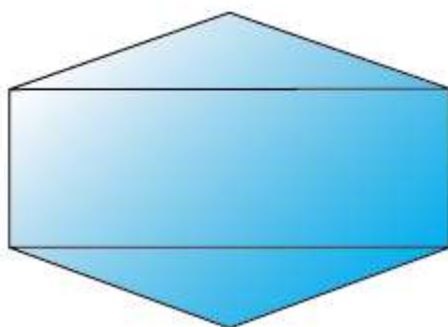
Area of Polygons

The area of any given polygon can be found by cutting the polygon into shapes whose area is known and adding the area of these shapes.

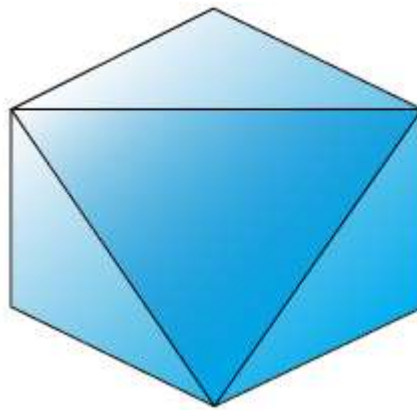
Some of the ways to find the area is shown below.



Area of this polygon = area of 2 trapeziums



Area of this polygon = Area of 2 triangles + Area of rectangle.



Area of this polygon = Area of 4 triangles.

Surface Area of Solids

Solid Shapes

Solid shapes or solid figures are the three-dimensional figures which have length, breadth and height. Using these, surface areas and volumes of these figures are found out.

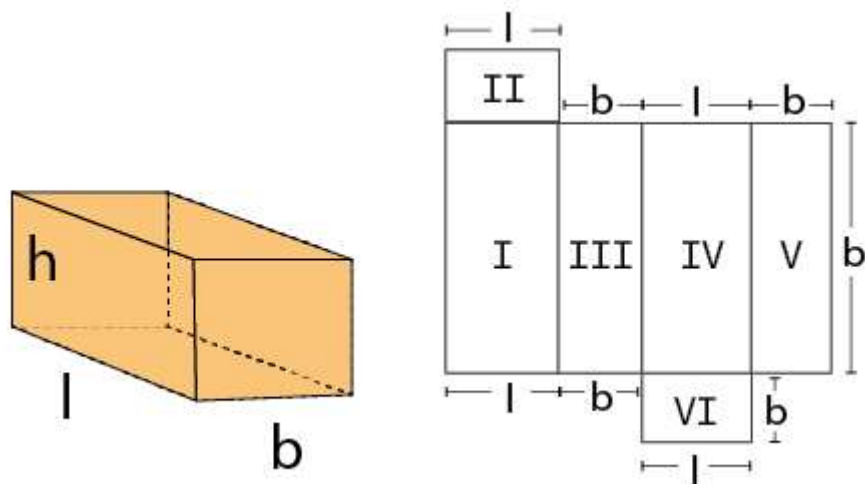
Solids with a Pair or More of Identical Faces



Surface Area of Solid Shapes

The surface area of the object is the total area occupied by the surface of the object.
or surface area is simply the sum of the areas of the flat surfaces (called faces).

Surface Area of a Cuboid

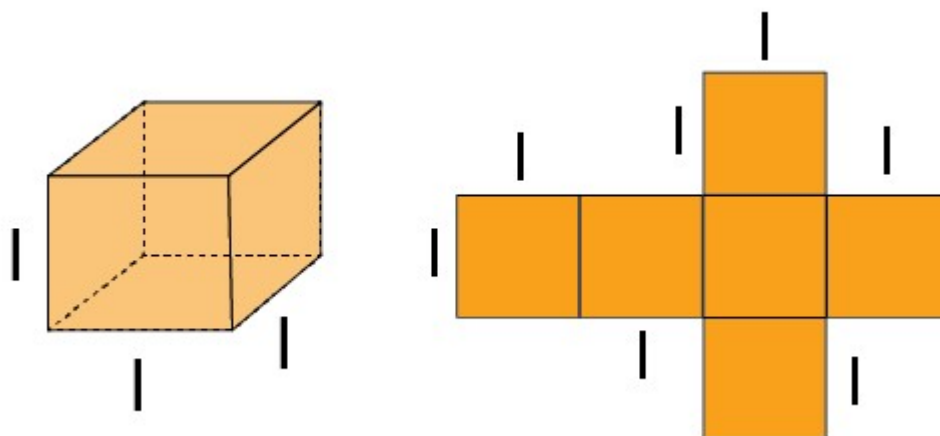


Total Surface area of cuboid = $2 (lb + bh + lh)$

Lateral Surface area of cuboid = $2h(l + b)$

Where, l is the length, b is the breadth and h is the height.

Surface Area of a Cube

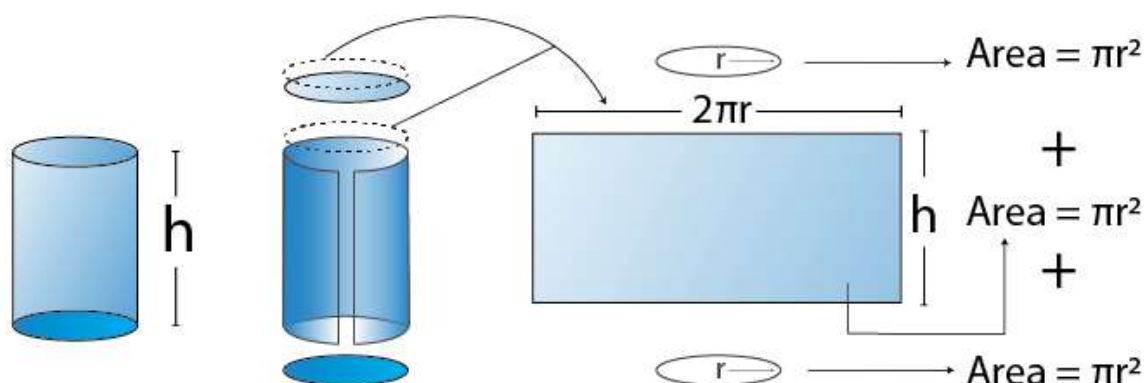


Total Surface area of a cube = $6l^2$

Lateral Surface area of a cube = $4l^2$

Where l is the length of each side of the cube.

Surface Area of a Cylinder



Curved surface area of cylinder (C.S.A) = $2\pi rh$

Total Surface area of cylinder (T.S.A) = $2\pi r(r + h)$

Where, r is the radius of the cylinder and h is the height of the cylinder.

Relation between Volume and Capacity

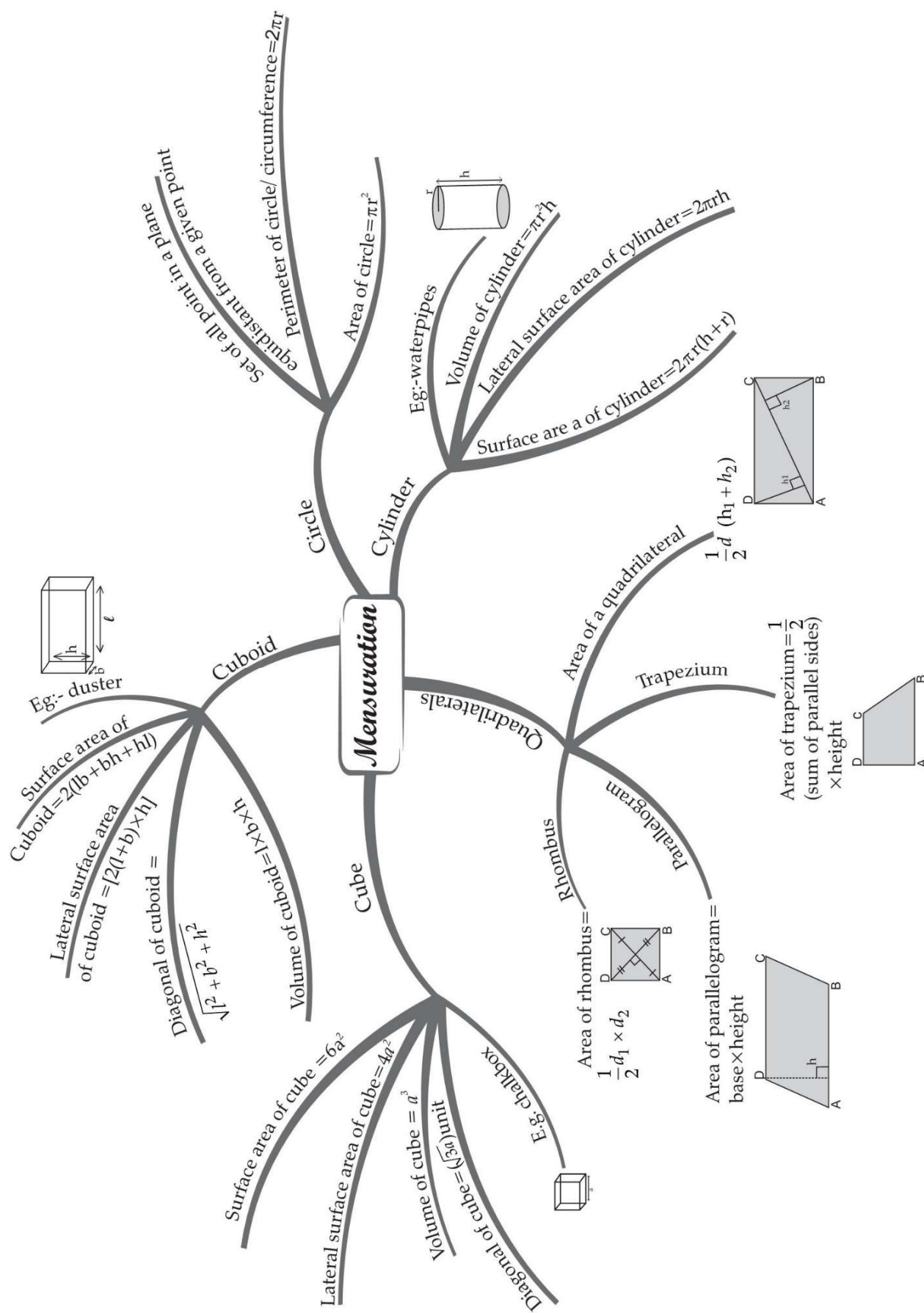
Volume is the total space occupied by an object. Volume is measured in cubic units,

Capacity refers to the maximum measure of an object's ability to hold a substance, like a solid, a liquid or a gas. Capacity can be measured in almost every other unit, including liters, gallons, pounds, etc.

E.g.: A bucket contains 9 litres of water, then its capacity is 9 litres.

MIND MAP : LEARNING MADE SIMPLE

CHAPTER-11



Important Questions

Multiple Choice Questions-

Question 1. The diagram has the shape of a



- (a) square
- (b) rectangle
- (c) triangle
- (d) trapezium.

Question 2. The diagram has the shape of a



- (a) rectangle
- (b) square
- (c) circle
- (d) parallelogram.

Question 3. The diagram has the shape of a



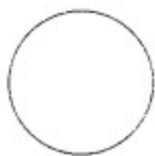
- (a) circle
- (b) rectangle
- (c) square
- (d) triangle.

Question 4. The diagram has the shape of a



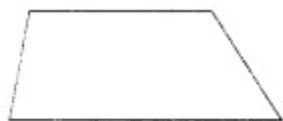
- (a) rectangle
- (b) square
- (c) parallelogram
- (d) circle.

Question 5. The diagram has the shape of a



- (a) circle
- (b) square
- (c) rectangle
- (d) parallelogram.

Question 6. The diagram has the shape of a



- (a) circle
- (b) parallelogram
- (c) rectangle
- (d) trapezium.

Question 7. The area of a rectangle of length a and breadth b is

- (a) $a + b$
- (b) ab
- (c) $a^2 + b^2$
- (d) $2ab$.

Question 8. The area of a square of side a is

- (a) a
- (b) a^2
- (c) $2a$
- (d) $4a$.

Question 9. The area of a triangle with base b and altitude h is

- (a) $\frac{1}{2}bh$
- (b) bh
- (c) $\frac{1}{3}bh$
- (d) $\frac{1}{4}bh$.

Question 10. The area of a parallelogram of base b and altitude h is

(a) $\frac{1}{2}bh$

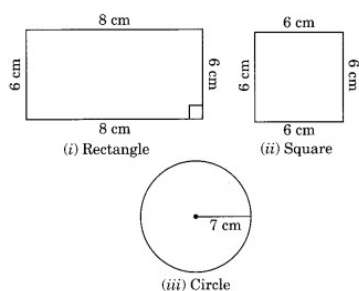
(b) bh

(c) $\frac{1}{3}bh$

(d) $\frac{1}{4}bh$.

Very Short Questions:

- Find the perimeter of the following figures:



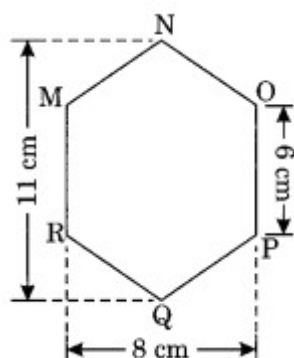
- The length and breadth of a rectangle are 10 cm and 8 cm respectively. Find its perimeter if the length and breadth are (i) doubled (ii) halved.
- A copper wire of length 44 cm is to be bent into a square and a circle. Which will have a larger area?
- The length and breadth of a rectangle are in the ratio 4 : 3. If its perimeter is 154 cm, find its length and breadth.
- The area of a rectangle is 544 cm^2 . If its length is 32 cm, find its breadth.
- If the side of a square is doubled then how much time its area becomes?
- The areas of a rectangle and a square are equal. If the length of the rectangle is 16 cm and breadth is 9 cm, find the side of the square.
- If the lengths of the diagonals of a rhombus are 16 cm and 12 cm, find its area.
- The area of a rhombus is 16 cm^2 . If the length of one diagonal is 4 cm, find the length of the other diagonal.
- If the diagonals of a rhombus are 12 cm and 5 cm, find the perimeter of the rhombus.

Short Questions :

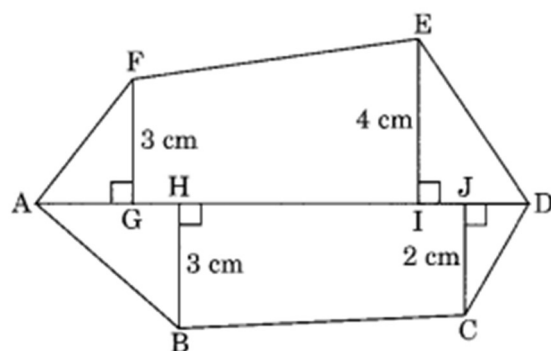
- The volume of a box is 13400 cm^3 . The area of its base is 670 cm^2 . Find the height of the box.
- Complete the following table; measurement in centimetres.

	(a)	(b)	(c)	(d)	(e)	(f)
Length	4	12	7	16	60	40
Breadth	5	8	6	—	—	24
Height	6	6	—	8	5	—
Volume	—	—	84	1536	5400	2400

- Two cubes are joined end to end. Find the volume of the resulting cuboid, if each side of the cubes is 6 cm.
- How many bricks each 25 cm by 15 cm by 8 cm, are required for a wall 32 m long, 3 m high and 40 cm thick?
- MNOPQR is a hexagon of side 6 cm each. Find the area of the given hexagon in two different methods.



- The area of a trapezium is 400 cm^2 , the distance between the parallel sides is 16 cm. If one of the parallel sides is 20 cm, find the length of the other side.
- Find the area of the hexagon ABCDEF given below. Given that: $AD = 8 \text{ cm}$, $AJ = 6 \text{ cm}$, $AI = 5 \text{ cm}$, $AH = 3 \text{ cm}$, $AG = 2.5 \text{ cm}$ and FG , BH , EI and CJ are perpendiculars on diagonal AD from the vertices F , B , E and C respectively.



- Three metal cubes of sides 6 cm, 8 cm and 10 cm are melted and recast into a big cube. Find its total surface area.

Long Questions :

- The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2 .

2. A rectangular metal sheet of length 44 cm and breadth 11 cm is folded along its length to form a cylinder. Find its volume.
3. 160 m^3 of water is to be used to irrigate a rectangular field whose area is 800 m^2 . What will be the height of the water level in the field?
4. Find the area of a rhombus whose one side measures 5 cm and one diagonal as 8 cm.
5. The parallel sides of a trapezium are 40 cm and 20 cm. If its non-parallel sides are both equal, each being 26 cm, find the area of the trapezium.
6. Find the area of polygon ABCDEF, if $AD = 18 \text{ cm}$, $AQ = 14 \text{ cm}$, $AP = 12 \text{ cm}$, $AN = 8 \text{ cm}$, $AM = 4 \text{ cm}$, and FM, EP, QC and BN are perpendiculars to diagonal AD.

Answer Key-

Multiple Choice questions-

1. (b) rectangle
2. (b) square
3. (d) triangle
4. (c) parallelogram
5. (a) circle
6. (d) trapezium
7. (b) ab
8. (b) a^2
9. (a) $\frac{1}{2}bh$
10. (b) bh

Very Short Answer :

1. (i) Perimeter of the rectangle = $2(l + b) = 2(8 + 6) = 2 \times 14 = 28 \text{ cm}$
 (ii) Perimeter of the square = $4 \times \text{side} = 4 \times 6 = 24 \text{ cm}$
 (iii) Perimeter of the circle = $2\pi r = 2 \times \frac{22}{7} \times 7 = 44 \text{ cm}$.
2. Length of the rectangle = 10 cm
 Breadth of the rectangle = 8 cm
 (i) When they are doubled,
 $l = 10 \times 2 = 20 \text{ cm}$
 and $b = 8 \times 2 = 16 \text{ cm}$
 Perimeter = $2(l + b) = 2(20 + 16) = 2 \times 36 = 72 \text{ cm}$
 (ii) When they are halved,

$$l = \frac{10}{2} = 5 \text{ cm}$$

$$b = \frac{8}{2} = 4 \text{ cm}$$

$$\text{Perimeter} = 2(l + b) = 2(5 + 4) = 2 \times 9 = 18 \text{ cm}$$

3. (i) When the wire is bent into a square.

$$\text{Side} = \frac{44}{4} = 11 \text{ cm}$$

$$\text{Area of the square} = (\text{side})^2 = (11)^2 = 121 \text{ cm}^2$$

- (ii) When the wire is bent into a circle.

$$\text{Circumference} = 2\pi r$$

$$44 = 2\pi r$$

$$44 = 2 \times \frac{22}{7} \times r$$

$$\therefore r = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm}$$

$$\therefore \text{Area of the circle} = \pi r^2$$

$$= \frac{22}{7} \times 7 \times 7$$

$$= 154 \text{ cm}^2$$

So, the circle will have a larger area.

4. Let the length of the rectangle be $4x$ cm and that of breadth = $3x$ cm

$$\text{Perimeter} = 2(l + b) = 2(4x + 3x) = 2 \times 7x = 14x \text{ cm}$$

$$14x = 154$$

$$x = 11$$

$$\text{Length} = 4 \times 11 = 44 \text{ cm}$$

$$\text{and breadth} = 3 \times 11 = 33 \text{ cm}$$

5. Area = 544 cm^2

$$\text{Length} = 32 \text{ cm}$$

$$\text{Breadth of the rectangle} = \frac{\text{Area}}{\text{Length}}$$

$$= \frac{544}{32}$$

$$= 17 \text{ cm}$$

6. Hence, the required breadth = 17 cm

Let the side of the square be x cm.

$$\text{Area} = (\text{side})^2 = x^2 \text{ sq. cm}$$

$$\text{If its side becomes } 2x \text{ cm then area} = (2x)^2 = 4x^2 \text{ sq. cm}$$

$$\text{Ratio is } x^2 : 4x^2 = 1 : 4$$

Hence, the area would become four times.

7. $\text{Area of the square} = \text{Area of the rectangle} = 16 \times 9 = 144 \text{ cm}^2$

$$\text{Side of the square} = \sqrt{\text{Area of the square}} = \sqrt{144} = 12 \text{ cm}$$

Hence, the side of square = 12 cm.

8. Given:

$$\text{First diagonal } d_1 = 16 \text{ cm}$$

$$\text{Second diagonal } d_2 = 12 \text{ cm}$$

$$\begin{aligned} \text{Area of the rhombus} &= \frac{1}{2} \times d_1 \times d_2 \\ &= \frac{1}{2} \times 16 \times 12 \\ &= 96 \text{ cm}^2 \end{aligned}$$

Hence, the required area = 96 cm².

9. Given: Area of the rhombus = 16 cm²

$$\text{Length of one diagonal} = 4 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area} &= \frac{1}{2} \times d_1 \times d_2 \\ 16 &= \frac{1}{2} \times 4 \times d_2 \\ \Rightarrow 16 \times 2 &= 4 \times d_2 \\ \Rightarrow 32 &= 4 \times d_2 \\ \therefore d_2 &= \frac{32}{4} = 8 \text{ cm} \end{aligned}$$

Hence, the required length = 8 cm.

10. Given: $d_1 = 12 \text{ cm}$, $d_2 = 5 \text{ cm}$

$$\begin{aligned} &= \frac{1}{2} \sqrt{(12)^2 + (5)^2} \\ &= \frac{1}{2} \sqrt{144 + 25} \\ &= \frac{1}{2} \sqrt{169} \\ &= \frac{1}{2} \times 13 \\ &= \frac{13}{2} \text{ cm} = 6.5 \text{ cm} \end{aligned}$$

The perimeter = $4 \times \text{side} = 4 \times 6.5 = 26 \text{ cm}$

Hence, the perimeter = 26 cm.

Short Answer :

1.

Volume of the box = 13400 cm^3

Area of the box = 670 cm^2

$$\begin{aligned}\text{Height} &= \frac{\text{Volume}}{\text{Base area}} \\ &= \frac{13400}{670} = 20 \text{ cm}\end{aligned}$$

Hence, the required height = 20 cm.

2.

$$\begin{aligned}(a) \quad V &= l \times b \times h \\ &= 4 \times 5 \times 6 = 120 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}(b) \quad V &= l \times b \times h \\ &= 12 \times 8 \times 6 = 576 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}(c) \quad V &= l \times b \times h \\ 84 &= 7 \times 6 \times h \\ \therefore h &= \frac{84}{7 \times 6} = 2 \text{ cm}\end{aligned}$$

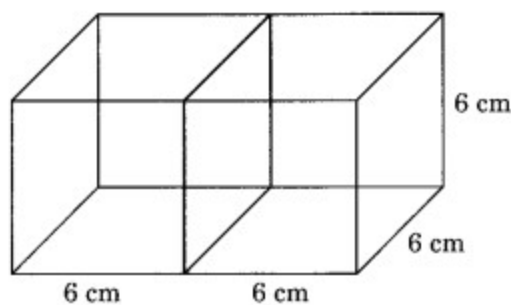
$$\begin{aligned}(d) \quad V &= l \times b \times h \\ 1536 &= 16 \times b \times 8 \\ \therefore b &= \frac{1536}{16 \times 8} = 12 \text{ cm}\end{aligned}$$

$$\begin{aligned}(e) \quad V &= l \times b \times h \\ 5400 &= 60 \times b \times 5 \\ \Rightarrow b &= \frac{5400}{60 \times 5} = 18 \text{ cm}\end{aligned}$$

$$\begin{aligned}(f) \quad V &= l \times b \times h \\ 2400 &= 40 \times 24 \times h \\ \therefore h &= \frac{2400}{40 \times 24} = 2.5 \text{ cm}\end{aligned}$$

Hence (a) \leftrightarrow 120 cm³, (b) \leftrightarrow 576 cm³, (c) \leftrightarrow 2 cm,
(d) \leftrightarrow 12 cm, (e) \leftrightarrow 18 cm, (f) \leftrightarrow 2.5 cm

3.



Length of the resulting cuboid = $6 + 6 = 12$ cm

Breadth = 6 cm

Height = 6 cm

Volume of the cuboid = $l \times b \times h = 12 \times 6 \times 6 = 432 \text{ cm}^3$

4. Converting into same units, we have,

Length of the wall = 32 m = $32 \times 100 = 3200$ cm

Breadth of the wall = 3 m = $3 \times 100 = 300$ cm

and the height = 40 cm

Length of the brick = 25 cm

breadth = 15 cm

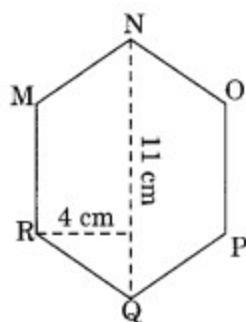
and height = 8 cm

Number of bricks required

$$\begin{aligned}
 &= \frac{\text{Volume of the wall}}{\text{Volume of one brick}} \\
 &= \frac{3200 \times 300 \times 40}{25 \times 15 \times 8} \\
 &= 128 \times 20 \times 5 = 12800
 \end{aligned}$$

Hence, the required number of bricks = 12800.

5. Method I: Divide the given hexagon into two similar trapezia by joining QN.



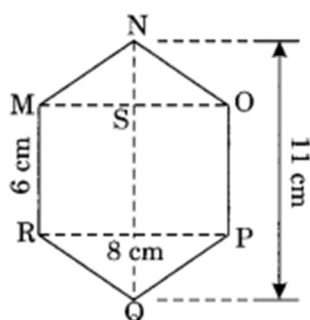
Area of the hexagon MNPQR = $2 \times$ area of trapezium MNQR

$$= 2 \times \frac{1}{2} (6 + 11) \times 4$$

$$= 17 \times 4$$

$$= 68 \text{ cm}^2$$

Method II: The hexagon MNOPQR is divided into three parts, 2 similar triangles and 1 rectangle by joining MO, RP.



$$\begin{aligned} NS &= \frac{11 \text{ cm} - 6 \text{ cm}}{2} \\ &= \frac{5}{2} \text{ cm} = 2.5 \text{ cm} \end{aligned}$$

$$\begin{aligned} \therefore \text{Area of hexagon MNOPQR} &= 2 \times \text{area of } \triangle MNO \\ &\quad + \text{area of rectangle MRPO} \\ &= 2 \times \left(\frac{1}{2} \times MO \times NS \right) + (RP \times MR) \\ &= MO \times NS + RP \times MR \\ &= 8 \times 2.5 + 8 \times 6 \\ &= 20 + 48 \\ &= 68 \text{ cm}^2. \end{aligned}$$

6. Given: Area of trapezium = 400 cm^2

Height = 16 cm

$$\text{Area of trapezium} = \frac{1}{2} (a + b) \times h$$

$$400 = \frac{1}{2} (20 + b) \times 16$$

$$\Rightarrow \frac{400 \times 2}{16} = 20 + b$$

$$\Rightarrow 50 = 20 + b$$

$$\therefore b = 50 - 20 = 30 \text{ cm}$$

Hence, the required length = 30 cm.

7. Given:

$$AD = 8 \text{ cm}$$

$$FG = 3 \text{ cm}$$

$$AJ = 6 \text{ cm}$$

$$EI = 4 \text{ cm}$$

$$AI = 5 \text{ cm}$$

$$BH = 3 \text{ cm}$$

$$AH = 3 \text{ cm}$$

$$CJ = 2 \text{ cm}$$

$$AG = 2.5 \text{ cm}$$

$$\text{Area of } \triangle AGF = \frac{1}{2} \times AG \times FG$$

$$= \frac{1}{2} \times 2.5 \times 3$$

$$= 2.5 \times 1.5$$

$$= 3.75 \text{ cm}^2$$

$$\text{Area of trapezium FGIE}$$

$$= \frac{1}{2} \times (GF + IE) \times GI$$

$$= \frac{1}{2} \times (3 + 4) \times 2.5 \quad [\because GI = AI - AG]$$

$$[\because GI = 5 - 2.5 = 2.5 \text{ cm}]$$

$$= \frac{1}{2} \times 7 \times 2.5$$

$$= 3.5 \times 2.5 = 8.75 \text{ cm}^2$$

$$\text{Area of } \triangle EID = \frac{1}{2} \times ID \times EI$$

$$= \frac{1}{2} \times (AD - AI) \times EI$$

$$= \frac{1}{2} \times (8 - 5) \times 4$$

$$= \frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$$

$$\text{Area of } \triangle CJD = \frac{1}{2} \times JD \times JC$$

$$= \frac{1}{2} \times (AD - AJ) \times JC$$

$$= \frac{1}{2} \times (8 - 6) \times 2$$

$$= \frac{1}{2} \times 2 \times 2 = 2 \text{ cm}^2$$

Area of trapezium HBCJ

$$\begin{aligned}
 &= \frac{1}{2} \times (\text{HB} + \text{JC}) \times \text{HJ} \\
 &= \frac{1}{2} \times (3 + 2) \times (\text{AJ} - \text{AH}) \\
 &= \frac{1}{2} \times 5 \times (6 - 3) \\
 &= \frac{1}{2} \times 5 \times 3 = 7.5 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of } \triangle \text{AHB} &= \frac{1}{2} \times \text{AH} \times \text{HB} \\
 &= \frac{1}{2} \times 3 \times 3 \\
 &= \frac{9}{2} = 4.5 \text{ cm}^2
 \end{aligned}$$

Area of hexagon ABCDEF = Area of $\triangle \text{AGF}$ + Area of trapezium FGIE + Area of $\triangle \text{EID}$ + Area of $\triangle \text{CJD}$ + Area of trapezium HBCJ + Area of $\triangle \text{AHB}$
 $= 3.75 \text{ cm}^2 + 8.75 \text{ cm}^2 + 6 \text{ cm}^2 + 2 \text{ cm}^2 + 7.5 \text{ cm}^2 + 4.5 \text{ cm}^2$
 $= 32.50 \text{ cm}^2$.

8. Volume of the cube with side 6 cm = $(\text{side})^3 = (6)^3 = 216 \text{ cm}^3$
 Volume of the cube with side 8 cm = $(\text{side})^3 = (8)^3 = 512 \text{ cm}^3$
 Volume of the cube with side 10 cm = $(\text{side})^3 = (10)^3 = 1000 \text{ cm}^3$
 Volume of the big cube = $216 \text{ cm}^3 + 512 \text{ cm}^3 + 1000 \text{ cm}^3 = 1728 \text{ cm}^3$
 Side of the resulting cube = $\sqrt[3]{1728} = 12 \text{ cm}$
 Total surface area = $6 (\text{side})^2 = 6(12)^2 = 6 \times 144 \text{ cm}^2 = 864 \text{ cm}^2$.

Long Answer :

1. Given: Diameter of the roller = 84 cm

$$\text{Radius} = \frac{84}{2} = 42 \text{ cm}$$

$$\text{Height} = 120 \text{ cm}$$

$$\text{Curved surface area of the roller} = 2\pi rh$$

$$\begin{aligned}
 &= 2 \times \frac{22}{7} \times 42 \times 120 \\
 &= 22 \times 1440 \\
 &= 31680 \text{ cm}^2 \\
 &= \frac{31680}{100 \times 100} \text{ m}^2 \quad [1 \text{ m}^2 = 10000 \text{ cm}^2] \\
 &= 3.168 \text{ m}^2
 \end{aligned}$$

Area covered by the roller in one complete revolution = 3.168 m^2

Area covered in 500 complete revolutions = $500 \times 3.168 = 1584 \text{ m}^2$

Hence, the required area = 1584 m^2 .

2. Volume of water = 160 m^3

Area of rectangular field = 800 m^2

Let h be the height of water level in the field.

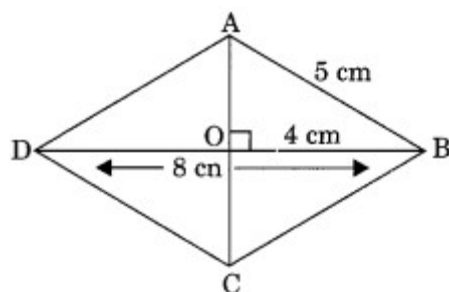
Now, the volume of water = volume of cuboid formed on the field by water.

$$160 = \text{Area of base} \times \text{height} = 800 \times h$$

$$\Rightarrow h = 0.2$$

So, required height = 0.2 m

3. Let ABCD be the rhombus as shown below.



$DO = OB = 4 \text{ cm}$, since diagonals of a rhombus are perpendicular bisectors of each other.

Therefore, using Pythagoras theorem in $\triangle AOB$, $AO^2 + OB^2 = AB^2$

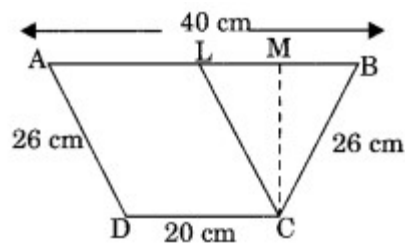
$$AO = \sqrt{AB^2 - OB^2} = \sqrt{5^2 - 4^2} = 3 \text{ cm}$$

$$\text{So, } AC = 2 \times 3 = 6 \text{ cm}$$

$$\begin{aligned}
 \text{Thus, the area of the rhombus} &= \frac{1}{2} \times d_1 \times d_2 \\
 &= \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2.
 \end{aligned}$$

4. Let ABCD be the trapezium such that

$AB = 40 \text{ cm}$ and $CD = 20 \text{ cm}$ and $AD = BC = 26 \text{ cm}$.



Now, draw $CL \parallel AD$

Then ALCD is a parallelogram.

So $AL = CD = 20$ cm

and $CL = AD = 26$ cm.

In $\triangle CLB$, we have $CL = CB = 26$ cm

Therefore, $\triangle CLB$ is an isosceles triangle.

Draw altitude CM of $\triangle CLB$.

Since $\triangle CLB$ is an isosceles triangle. So, CM is also the median.

Then $LM = MB = \frac{1}{2}BL = \frac{1}{2} \times 20 \text{ cm} = 10 \text{ cm}$

[as $BL = AB - AL = (40 - 20) \text{ cm} = 20 \text{ cm}$].

Applying Pythagoras theorem in $\triangle CLM$, we have

$$CL^2 = CM^2 + LM^2$$

$$26^2 = CM^2 + 10^2$$

$$CM^2 = 26^2 - 10^2 = (26 - 10)(26 + 10) = 16 \times 36 = 576$$

$$CM = \sqrt{576} = 24 \text{ cm}$$

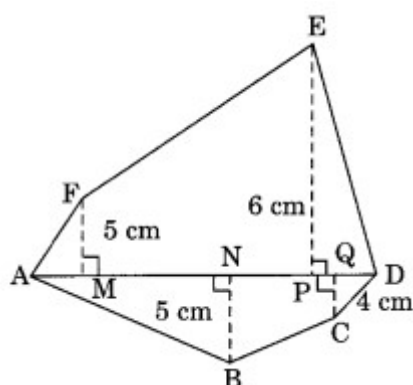
Hence, the area of the trapezium $= \frac{1}{2} (\text{sum of parallel sides}) \times \text{height}$

$$= \frac{1}{2} (20 + 40) \times 24$$

$$= 30 \times 24$$

$$= 720 \text{ cm}^2.$$

5.



In the figure

$$MP = AP - AM = (12 - 4) \text{ cm} = 8 \text{ cm}$$

$$PD = AD - AP = (18 - 12) \text{ cm} = 6 \text{ cm}$$

$$NQ = AQ - AN = (14 - 8) \text{ cm} = 6 \text{ cm}$$

$$QD = AD - AQ = (18 - 14) \text{ cm} = 4 \text{ cm}$$

Area of the polygon ABCDEF = area of $\triangle AFM$ + area of trapezium FMPE + area of $\triangle EPD$ + area of $\triangle ANB$ + area of trapezium NBCQ + area of $\triangle QCD$.

$$= \frac{1}{2} \times AM \times FM + \frac{1}{2} (FM + EP) \times MP + \frac{1}{2} PD$$

$$\times EP + \frac{1}{2} \times AN \times NB + \frac{1}{2} (NB + CQ) \times NQ$$

$$+ \frac{1}{2} QD \times CQ$$

$$= \frac{1}{2} \times 4 \times 5 + \frac{1}{2} (5 + 6) \times 8 + \frac{1}{2} \times 6 \times 6 + \frac{1}{2}$$

$$\times 8 \times 5 + \frac{1}{2} (5 + 4) \times 6 + \frac{1}{2} \times 4 \times 4.$$

$$= 10 + 44 + 18 + 20 + 27 + 8 = 127 \text{ cm}^2$$